Crown Ratio as a Proxy for Vigor in Stand-Level Basal Area Models

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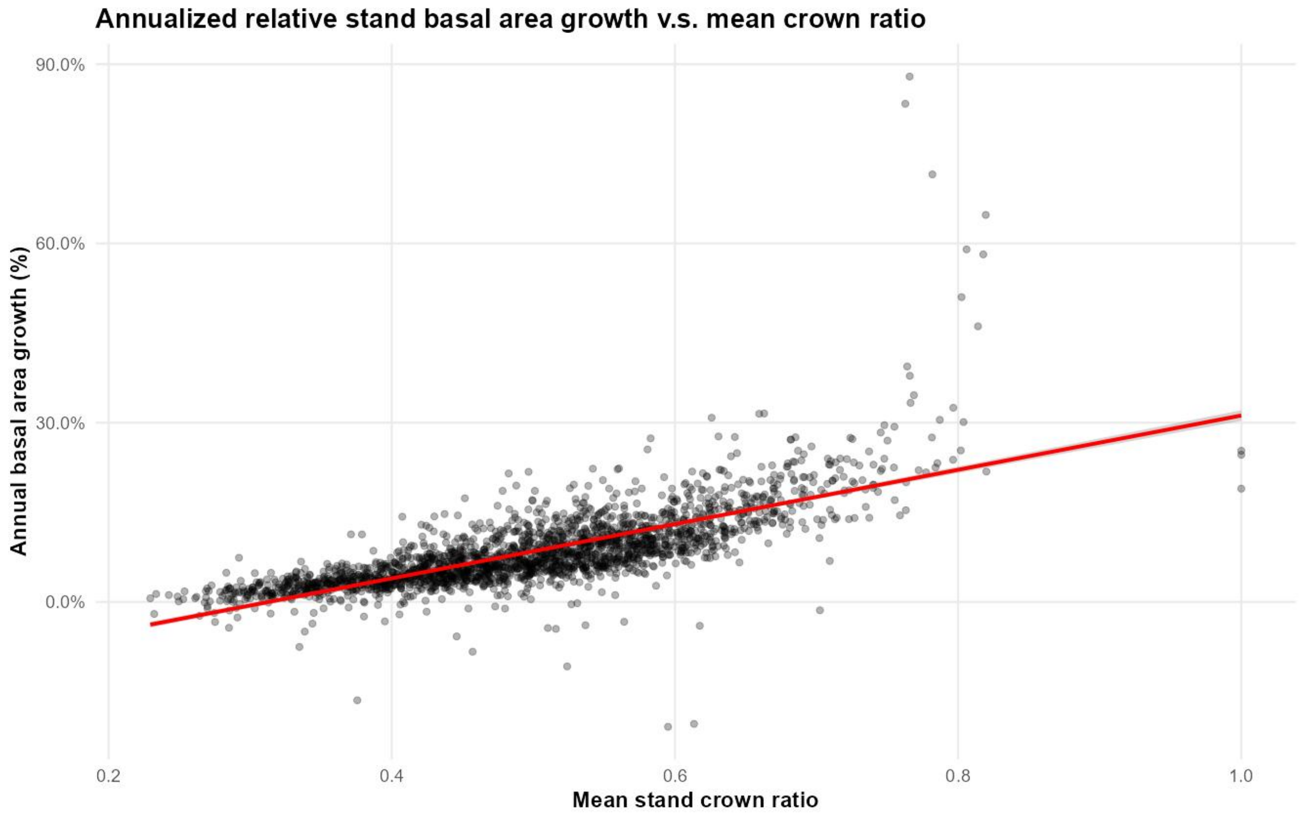
Topics for today

- Inspiration for research
- General relationship between crown ratio and growth
- Exploring crown ratio as a predictor in stand-level basal area projection
- Development of models
- Conclusions

Inspiration

- Main research involves individual-tree growth and yield systems
  - Crown ratio is a main driver of individual tree growth

- 1/3rd crown ratio "rule of thumb" for thinning loblolly pine plantations
  - Can addition of a crown ratio term increase predictive accuracy post-thinning?



Guiding Questions

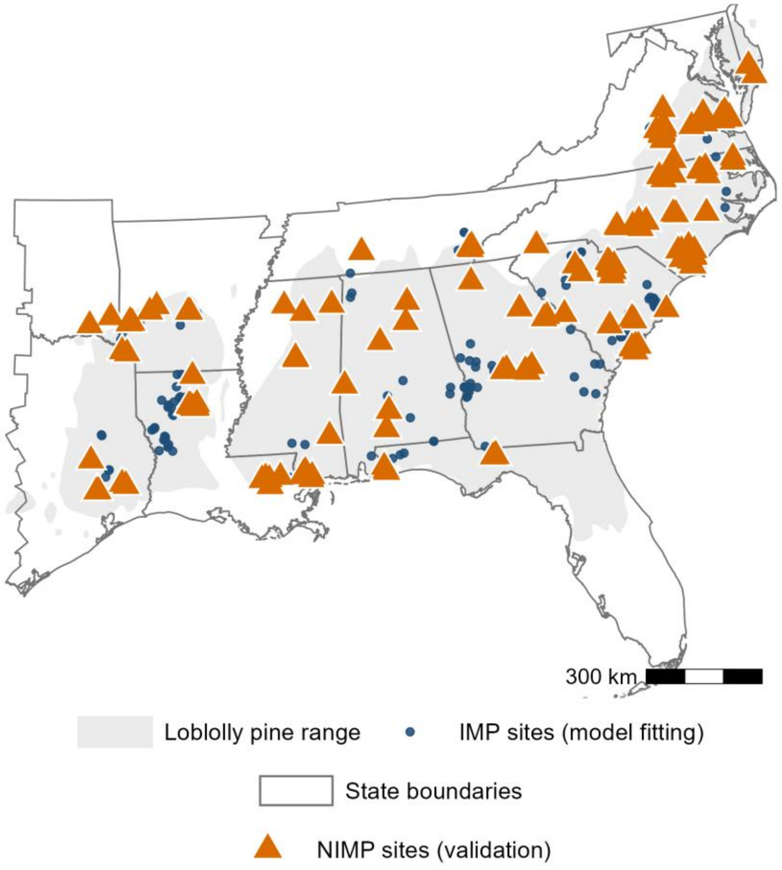
- Will an average tree crown ratio term benefit a stand-level model?
  - Is the average crown ratio the appropriate term to use?
- How does the basal area model perform without a thinning response variable (particularly one which requires a basal area before thinning)?
- Can an alternative thinning response be introduced that does not require basal area before thinning?
- Is there an interaction between average stand crown ratio and thinning?
- Should years since thinning be considered along with a regular indicator variable stating a thinning has occurred?

A basal area projection equation

- A basal area projection model developed and extensively studied by the Forest Modeling Research Cooperative
  - Current iteration implemented within the stand-level growth and yield system, FASTLOB
- $$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0}_{\beta_3\frac{H_{dt}}{H_{d2}}} \beta_1 S^{\beta_2} \left(\frac{G_a}{G_b}\right)^{\beta_3\frac{H_{dt}}{H_{d2}}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$
  $$where\ TR = \left(\frac{G_a}{G_b}\right)^{\beta_3\frac{H_{dt}}{H_{d2}}}$$
- Intensively managed plantation (IMP) dataset used to fit
- Non-intensively managed plantation (NIMP) dataset used to validate

Study Data Locations

IMP sites were used for model fitting;
NIMP sites were used for validation



Data source: Forest Modeling Research Cooperative (FMRC)

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Intensively managed plantation dataset

- 170 permanent installations were established during the dormant seasons between 1996-2000 in 3- to 8-year-old intensively managed plantations.
- Each plot received site preparation, vegetation control treatments appropriate for site and were planted with genetically improved stock.
- Some of the measurements taken: total height (nearest foot), height to live crown (nearest foot), DBH (nearest 0.1-inch), crown class, species, stem quality assessment.
- Each plot was randomly assigned one of three treatments: a control (no thin), a light thinning (residual basal area of 70-85 sq ft / acre), or heavy thinning (residual basal area of 50-65 sq ft / acre).
- Measured on a 2-year cycle

Non-intensively managed plantation dataset

- Established during dormant seasons of 1980-1981 and 1981-1982 in cutover, site-prepared plantations ranging in age from 8 to 25 years
- Chosen plantations were un-thinned, un-pruned, free from interplanting, contain no ice or wind damage, insect or disease presence, or prior fertilization within 4 years.
- Required minimum of 200- 300 planted stems per acre with no more than 25% of volunteer pines in the main canopy
- Some of the measurements taken: total height (nearest foot), height to live crown (nearest foot), DBH (nearest 0.1-inch), crown class, species, stem quality assessment.
- Each plot was randomly assigned one of three treatments: a control (no thin), a light thinning (residual ba of 70-85 sq ft / acre), or heavy thinning (residual ba of 50-65 sq ft / acre).
- Plots were measured on 3-year intervals

Obtaining crown ratio for use

- Differing ways of obtaining individual tree crown ratio for stand-level crown ratio attribute:
    - Manual measurement of height to live crown
    - Direct prediction of crown ratio given sampled tree dimensions
        - Many empirical models have been developed for height to crown base or crown ratio for loblolly pine
        - Predicting crown ratio, height to crown base, fixed effects verses mixed effects?
    - Remotely sensed derived height to live crown?
- How do we aggregate the individual tree metric to work with a stand-level model?

Implementation of crown ratio in a stand-level model

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Initial Experimentation: Basal Area Projection

$$(1)\ \ G_2 = G_1^{\frac{HD_1}{HD_2}} exp\left\{\left[\frac{G_1}{G_t}\right]^{b_0} b_1 S^{b_2} \left(\frac{G_a}{G_b}^{b_3\left(\frac{HD_t}{HD_2}\right)}\right)\left[1-\frac{HD_1}{HD_2}\right]\right\}$$

$$(2)\ \ G_2 = G_1^{\frac{HD_1}{HD_2}} exp\left\{\left[\frac{G_1}{G_t}\right]^{b_0} b_1 S^{b_2} \phi(CR) \left(\frac{G_a}{G_b}^{b_3\left(\frac{HD_t}{HD_2}\right)}\right)\left[1-\frac{HD_1}{HD_2}\right]\right\}$$

$$(3)\ \ G_2 = G_1^{\frac{HD_1}{HD_2}} exp\left\{\left[\frac{G_1}{G_t}\right]^{b_0} b_1 S^{b_2} \left(\frac{G_a}{G_b}^{b_3\left(\frac{HD_t}{HD_2}\right)\phi(CR)}\right)\left[1-\frac{HD_1}{HD_2}\right]\right\}$$

$$(4)\ \ G_2 = G_1^{\frac{HD_1}{HD_2}} exp\left\{\left[\frac{G_1}{G_t}\right]^{b_0} b_1 S^{b_2} \phi(CR) \left(\frac{G_a}{G_b}^{b_3\left(\frac{HD_t}{HD_2}\right)\phi(CR)}\right)\left[1-\frac{HD_1}{HD_2}\right]\right\}$$

Where the effect of crown ratio on the base model and thinning response was attempted using four different terms:

- $\phi(CR) = \beta * CR$
- $\phi(CR) = (1 + \beta * CR)$
- $\phi(CR) = exp^{\beta * CR}$
- $\phi(CR) = exp^{\beta(CR - CR_{ref})}$

Crown ratio reflects vigor, light availability, and live-crown efficiency and is often used in individual tree models

Attempted to allow average crown structure to modulate growth efficiency (in baseline rate and thinning response)

- Adding $\phi(CR)$ to base term: stands with larger live crown ratios result in more basal area growth
- Adding $\phi(CR)$ to thinning response: thinning intensity may result in different responses based on remaining trees' crown health

Evaluated Crown Ratio Terms

Linear crown ratio effect terms proved difficult to fit in the base projection model (minor success in thinning response), so the nonlinear effect terms proved preferable.

- $\phi(CR) = \beta * CR$
- $\phi(CR) = (1 + \beta * CR)$
- $\phi(CR) = e^{\beta * CR}$
- $\phi(CR) = e^{\beta(CR - CR_{ref})}$
- The term, $e^{\beta * CR}$, proved to be the easiest to fit and adequate for inclusion in the equation

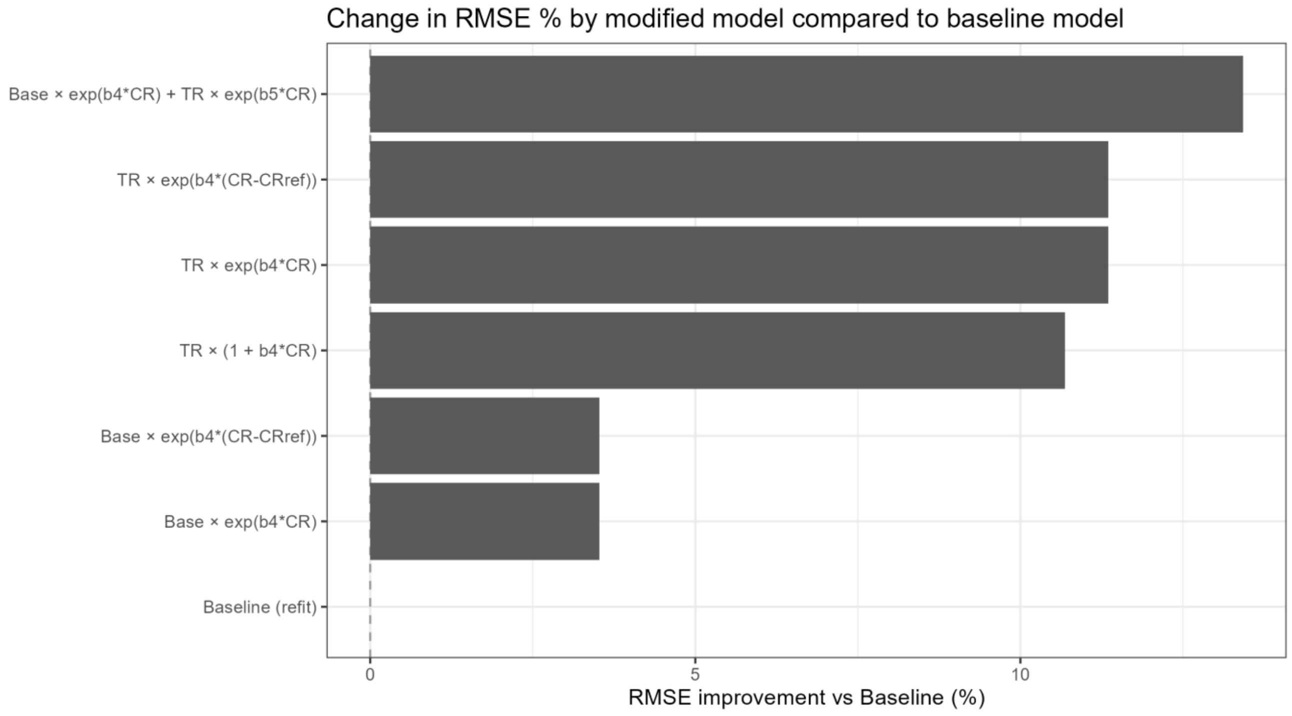
Expected patterns:

- Higher CR values (~0.6 – 0.7) represent a stand with vigorous trees and little suppression = faster basal area growth and strong thinning response
- Lower CR values (~0.2 – 0.4) represent a stand with stressed or shaded trees = slower recovery and reduced response to thinning

By including CR, the basal area projection model is adaptive to average physiological state of trees and not just a function of stand structure (BA/acre, HD, SI)

Table 2: FASTLOB basal area projection model improvement

Model	n	RMSE	MAE	Bias	ΔAICc	Improvement vs Base (RMSE%)
Base x exp(b4*CR) + TR x exp(b5*CR)	1677	6.75	4.728	0.31	0	13.42
TR x exp(b4*(CR-CRref))	1677	6.911	4.777	0.132	77.25	11.35
TR x exp(b4*CR)	1677	6.911	4.777	0.132	77.25	11.35
TR x (1 + b4*CR)	1677	6.963	4.775	0.135	102.34	10.69
Base x exp(b4*(CR-CRref))	1677	7.522	5.116	0.298	361.13	3.52
Base x exp(b4*CR)	1677	7.522	5.116	0.298	361.13	3.52
Baseline (refit)	1677	7.796	5.166	0.095	479.35	0



Takeaways from addition of crown ratio to model

Incorporating average crown ratio into the basal area projection model improved fit by 11–13% (Training RMSE reduction) relative to the baseline

The best model allowed CR to modify both the base growth projection and thinning response

- This supports our hypothesis that crown vigor influences both inherent productivity and release response.

This suggests further testing is required

- Validation needed to determine predictive performance

Basal Area Projection Equation (FASTLOB)

1) Original FASTLOB basal area projection equation

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right] \quad where\ TR = \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}}$$

2) Stripped down equation without thinning response

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

3) Stripped down equation with crown ratio term replacing thinning response

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

4) FASTLOB with thinning response and crown ratio term

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

5) FASTLOB with crown ratio modifying base function and thinning response

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}} \phi(CR)} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

6) FASTLOB with indicator thinning response dependent on average crown ratio

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \exp((\beta_3 + \beta_4 T)CR) \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

7) FASTLOB with indicator thinning response dependent on average crown ratio and years since thinning input (exponential decay response = strongest effect immediately after thinning)

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \exp((\beta_3 + \beta_4 T)CR)\, e^{\gamma_1 e^{(-\gamma_2 YST)T}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

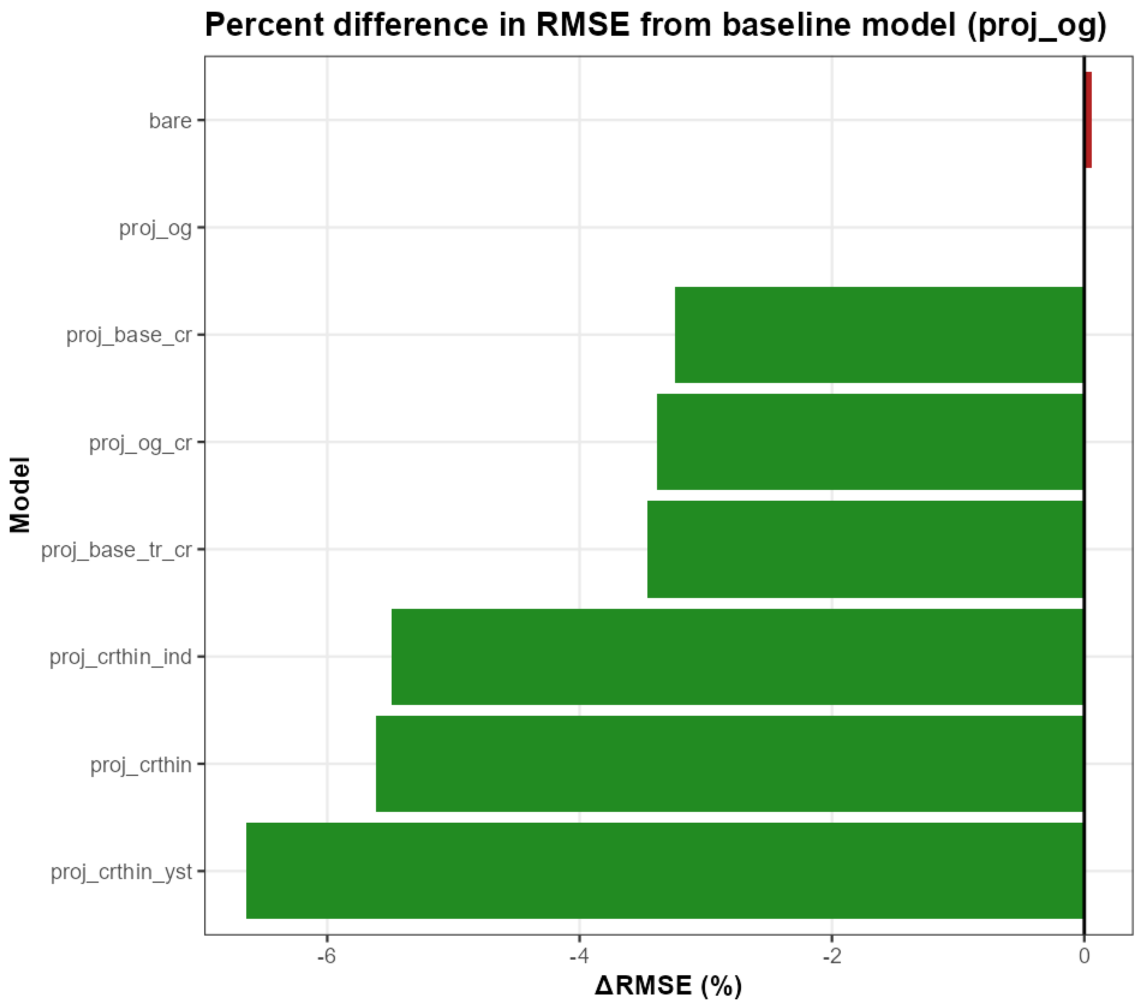
8) FASTLOB with years since thinning input but independent simple thinning and crown ratio effect

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} e^{(\beta_3 CR + \beta_4 T \gamma_1 e^{(-\gamma_2 YST)T})} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

These models were fit individually using IMP data and validated using NIMP data.

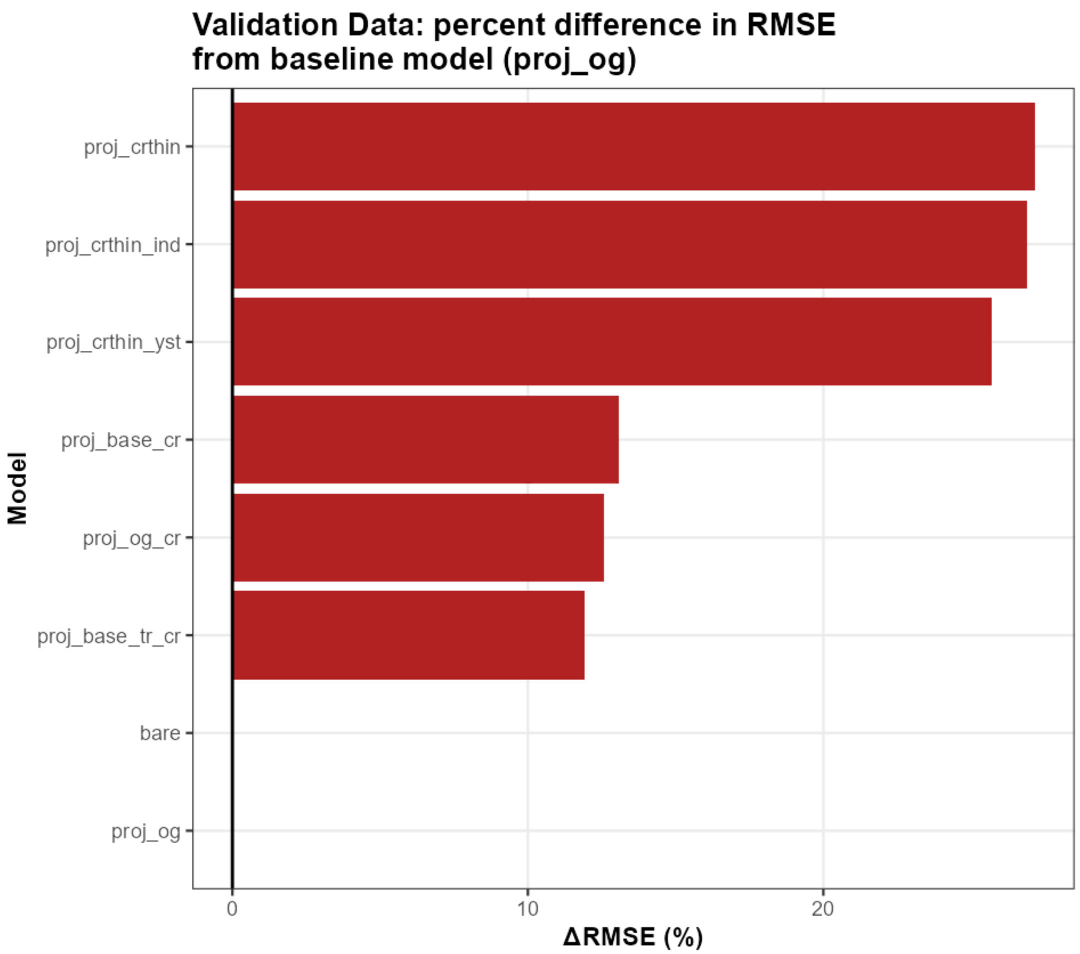
- Swapping training and test data resulted in similar trends

Change in Training RMSE Relative to Base Model



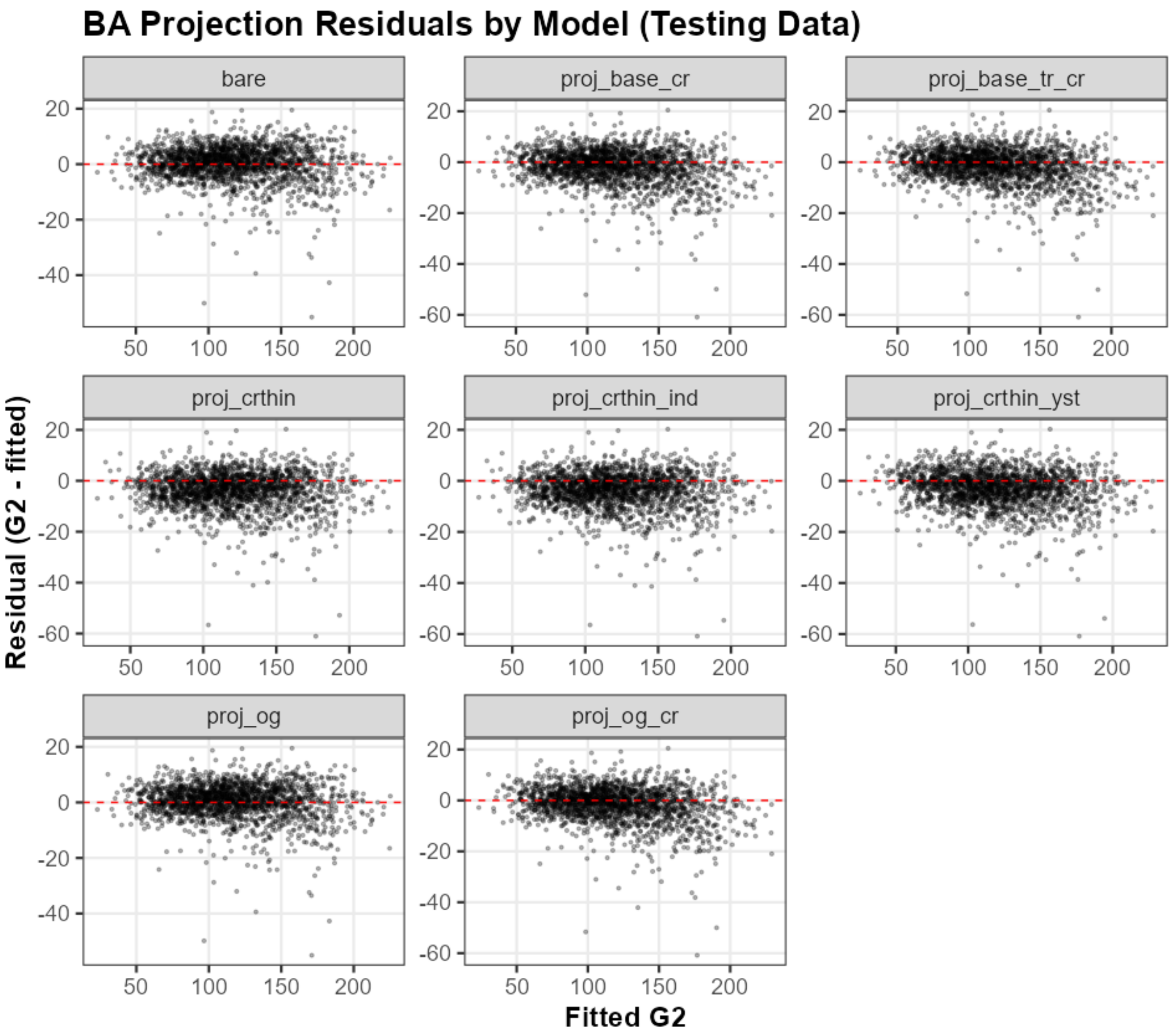
- Earlier work fit and compared models using NIMP data
- IMP data is now preferred to fit these models as it spans broader ages and stand conditions
- After cleaning, ΔRMSE relative to a base model is computed
- Current figure does not show performance given independent dataset but compares relative efficiency of varied functional forms given same information
  - Still must be validated with an independent dataset to know out-of-sample predictive performance

Change in Validation RMSE Relative to Base Model



- Validation with NIMP data indicates that the base and stripped-down model forms perform best out-of-sample
- The opposing result from the training RMSE suggests that added structural complexity may not generalize beyond the training data
  - Added terms improve local fit but provide no robust benefit
- Additional parameters may be capturing noise or dataset-specific structure rather than stable signal
- Further evaluation of added terms under broader validation scenarios?

Residual Plots – Validation Data



Basal Area Projection Equation (FASTLOB)

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right] \quad where\ TR = \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}}$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \phi(CR) \left(\frac{G_a}{G_b}\right)^{\beta_3 \frac{H_{dt}}{H_{d2}} \phi(CR)} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \exp((\beta_3 + \beta_4 T) CR) \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} \exp((\beta_3 + \beta_4 T) CR)\, e^{\gamma_1 e^{(-\gamma_2 YST) T}} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

$$G_2 = G_1^{\frac{H_{d1}}{H_{d2}}} \exp\left[\left(\frac{G_1}{G_t}\right)^{\beta_0} \beta_1 S^{\beta_2} e^{\left(\beta_3 CR + \beta_4 T \gamma_1 e^{(-\gamma_2 YST) T}\right)} \left(1 - \frac{H_{d1}}{H_{d2}}\right)\right]$$

Table: FASTLOB basal area projection model performance (with validation dataset)

Model	n	RMSE	MAE	Bias	AICc	BIC	% Difference in RMSE from Original Model
proj_og	1901	6.349	4.547527	0.332195	7035.3	7057.4	0.00%
bare	1901	6.35	4.5378	0.193822	7034.1	7050.7	0.02%
proj_base_cr	1901	7.179	5.033134	-1.84738	7502.2	7524.4	13.07%
proj_og_cr	1901	7.148	5.018418	-1.65104	7487.7	7515.4	12.58%
proj_base_tr_cr	1901	7.106	4.999344	-1.54511	7467.6	7500.9	11.92%
proj_crthin	1901	8.073	5.809958	-3.7889	7950.4	7978.1	27.15%
proj_crthin_yst	1901	7.979	5.768439	-3.23718	7910.3	7949.1	25.68%
proj_crthin_ind	1901	8.056	5.764235	-3.72086	7946.6	7985.4	26.89%

Applicability of crown ratio in stand-level models?

Conclusions

Crown ratio and thinning modifiers improved in-sample fit but did not generalize well.

Validation performance declined as model complexity increased:

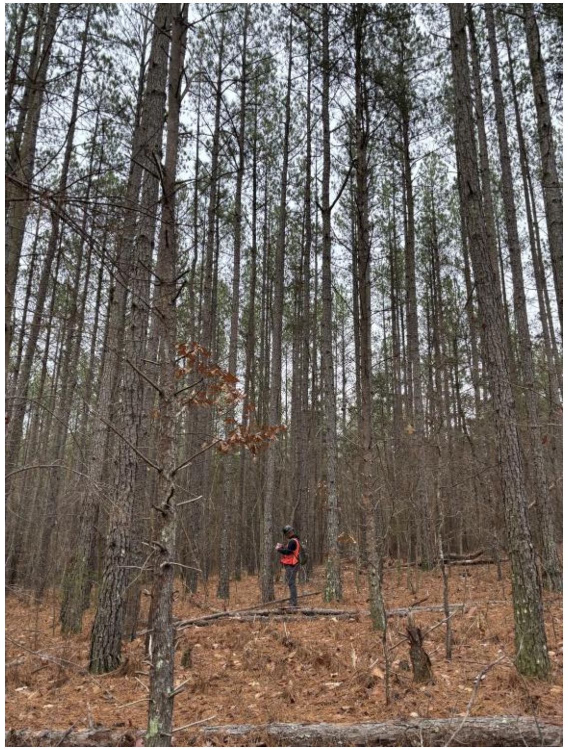
- Higher RMSE, AIC, BIC, absolute bias relative to original model.
- Indicates overfitting and reduced robustness.

Why might this be happening?

- Crown ratio is weak at the stand level
  - Highly variable and generalized
- Core drivers already capture most variation:
  - Basal area (total and pine), dominant height (observed and projected), site index
  - Crown ratio becomes partially redundant

Acknowledgements

- Corey Green
- Forest Modeling Research Cooperative
  - Member companies
- Fellow FREC graduate students



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Questions?

References

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